Kineton High School & Sixth Form

Achieving Personal Best

Department of Mathematics

Introduction to A-Level Mathematics

Congratulations on your choice to study Mathematics at A-level! Mathematics at A-level is both a demanding and rewarding course. The Mathematics Department at Kineton High School is committed to ensuring that you make good progress throughout your A-level course. To ensure that you make the best possible start to the course, we have prepared this booklet.

It is <u>vitally important</u> that you spend time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics <u>before</u> you commence your course in September. You should have met all the topics before at GCSE (there is nothing new here!). There are 6 sections in this booklet covering what we deem to be the basic skills required to start your A-level course on the best footing. As a result, if you find that you are struggling with a particular section or set of questions, you must seek to independently review this area of mathematics using the sources of information detailed in this booklet or your own GCSE notes/ revision guides/web resources. For each topic there are video numbers – these correspond to videos on the Corbett Maths website (www.corbettmaths.com/contents) which has help videos to explain each topic in detail.

During your first maths lesson in September, the work completed from this booklet must be handed to Mr Moulder.

Any pupil who has not completed this work over the summer, having had the opportunity to do so, is unlikely to have the level of commitment required in order to succeed in A-level maths.

We hope that you will use this introduction to give you a good start to your A-level work and that it will help you enjoy and benefit from the course.

You may find the following websites useful:

https://www.corbettmaths.com/contents
Revision videos (specific video numbers given at the start of each topic in this booklet)

http://www.hegartymaths.com/
Revision videos

http://www.themathsteacher.com/index.php
Revision videos and worksheets

http://www.bbc.co.uk/schools/gcsebitesize/maths/
Information, videos and online tests

http://www.khanacademy.org/exercisedashboard
Revision videos and online questions

CONTENTS

1. Quadratics (videos 10, 118, 119, 120, 266, 267 & 267a)

You need to be confident when factorising quadratic expressions and solving quadratic equations. Some of the methods that you may have been introduced to during your GCSE course include:

- Factorising expressions of the form $ax^2 + bx + c$ (for cases where a = 1 <u>and</u> a ≠ 1)
- Completing the square
- The quadratic formula
- Difference of two squares.

The following questions require application of the above skills.

<u>Section 1</u>

Factorise the following:

a) $x^2 + 8x + 15$
b) $x^2 + 8x + 7$
c) $x^2 + 9x + 20$
d) $x^2 - 6x + 5$
e) $x^2 - 2x + 1$
f) $x^2 + 3x - 18$
g) $x^2 - x - 12$

<u>Section 2</u>

Factorise the following:

a) $x^2 - 100$
b) $x^2 - 36$
c) $w^2 - 2500$
d) $y^2 - 144$
e) $10,000 - a^2$
f) $(x+1)^2 - 4$

Section 3

Factorise the following:

a) $5x^2 + 16x + 3$

b) $2x^2 + 11x + 5$

c) $3x^2 + 4x + 1$

d) $8x^2 + 6x + 1$

e) $6x^2 + 13x + 6$

f) $5x^2 - 7x + 2$

g) $3x^2 - 10x - 8$

h) $2x^2 + 7xy + 5y^2$

Section 4

Complete the square on the following:

a) $x^2 + 4x + 5$

b) $x^2 + 8x + 17$

c) $x^2 + 10x - 20$

d) $x^2 - 6x + 11$

e) $x^2 - 20x + 80$

f) $x^2 - 26x - 1$

g) $x^2 - x + 1$

h) $x^2 + 5x - 5$

Section 5

Use any of the above methods of factorisation or the quadratic formula to solve the following quadratics. Where necessary, give answers to 3 significant figures.

a) $2x^2 - 108 = 0$

b) $2k^2 - 11k + 5 = 0$

c) $4m^2 - 4m = 3$

d) $(2n - 1)(3n + 2) = 24$

e) $x^2 - 5x - 8 = 0$

f) $10x^2 - 5x - 4 = 0$

g) $2w^2 + 5w - 3 = 0$

h) $3(x + 2)^2 - 3x(x + 2) = 0$

2. Simultaneous equations (videos 295 & 298)

At GCSE you will have met two methods of solving simultaneous equations (elimination and substitution). You must know how to use the substitution method of solving simultaneous equations as this method can also be used for solving non-linear simultaneous equations.

Section 1

Solve the following linear simultaneous equations:

a) $8x + 3y = 57$ and $6x + 3y = 51$
b) $6x + 7y = 32$ and $11x + 7y = 47$
c) $12x – 4y = 132$ and $20x – 4y = 228$
d) $3x – y = 30$ and $3x + 2y = 21$
e) $12y – 7x = -77$ and $-8y – 7x = -77$
f) $4x – 3y = 14$ and $2x + 2y = -7$
g) $3x + 2y = 11$ and $2x - 5y = 1$
h) $5x + 4y = 5$ and $3x – 5y = -34$
i) $7x - 2y = 13$ and $4x - 3y = 13$
j) $4x - 3y = 5$ and $2x + 2y = -1$

Section 2

Solve the following simultaneous equations (one is linear and one is quadratic):

a) $x^2 + y = 6$ and $y = x$
b) $x^2 – 2y = 2$ and $y = x + 3$
c) $x^2 + 4y = 7$ and $2y + x = 2$
d) $y = 3x^2 – 2$ and $y = 3 – 2x$
e) $y = 3 – x^2$ and $y = 5 – 3x$
f) $2y = 4x^2 – 7$ and $y = 6x$

3. Indices (videos 173, 174 & 175)

At GCSE, you will have dealt with different types of powers including positive integers, fractional and negative powers.

Section 1

Re-write the following in fractional form (that do not contain indices), without using a calculator:

a) $4^{-2} =$ b) $6^{-3} =$ c) $9^{\frac{1}{2}} =$

d) $27^{\frac{1}{3}} =$ e) $16^{\frac{3}{4}} =$ f) $4^{\frac{5}{2}} =$

Section 2

Find the value of x, without using a calculator:

a) $3^x = \frac{1}{81}$ b) $17^x = \frac{1}{17}$ c) $125^x = 5$

d) $a^x = \frac{1}{a^3}$ e) $\sqrt{m} = m^x$ f) $\frac{1}{p^2} = p^x$

g) $\sqrt[3]{q} = q^x$ h) $\sqrt[3]{q^2} = q^x$ i) $\sqrt[5]{q^2} = q^x$

Section 3

a) Express $81^{-\frac{1}{2}}$ as a fraction in the form $\frac{a}{b}$, where a and b are integers.
b) Simplify $a^6 \div a^2$
c) Find the value of y for which $2 \times 4^y = 64$

4. Algebraic fractions (videos 21 & 24)

Work done on adding, subtracting, multiplying and dividing numerical fractions is built upon at A-level through the use of algebraic fractions. You will need skills from the previous section in order to answer the following questions.

<u>Section 1</u>
Simplify fully:

a) $\dfrac{x^2+3x}{x+3}$

b) $\dfrac{2x-4x^2}{2x-1}$

c) $\dfrac{x^2+4x+3}{x^2+5x+6}$

d) $\dfrac{2x^2-8}{x^2+4x+4}$

e) $\dfrac{2x^2+5x+3}{3x^2+5x+2}$

f) $\dfrac{16-x^2}{x-4}$

<u>Section 2</u>
Write as a single fraction in its simplest form:

a) $\dfrac{3}{10x}+\dfrac{4}{10x}$

b) $\dfrac{x}{5}+\dfrac{2x}{15}$

c) $\dfrac{x}{2}-\dfrac{x}{8}$

d) $\dfrac{1}{x+2}+\dfrac{1}{x+3}$

e) $\dfrac{4}{x+2}-\dfrac{3}{x+1}$

f) $\dfrac{1}{2x-1}-\dfrac{1}{2x+3}$

g) Factorise x^2+3x+2

Write $\dfrac{1}{x+2}+\dfrac{1}{x^2+3x+2}$ as a single fraction in its simplest form.

h) Factorise $2x^2-3x+1$

Write $\dfrac{2}{2x-1}+\dfrac{1}{2x^2-3x+1}$ as a single fraction in its simplest form.

i) Write $\dfrac{1}{2x+6}+\dfrac{1}{x^2+4x+3}$ as a single fraction in its simplest form.

5. Straight lines (videos 12, 87, 88, 191, 194, 195, 196 & 197)

From GCSE you should be familiar with:
- Finding the midpoint of a line segment
- The gradient and y intercept of a straight line
- The equation of a straight line ($y = mx + c$)
- The relationships between parallel and perpendicular lines
- Finding the distance between two points/length of a line segment.

The following exercise will review some of these skills.

Section 1

Work out the midpoint of the line joining:

a) (2, 3) and (7, 11)
b) (0, 0) and (2, 4)
c) (-3, 5) and (0, 1)
d) (-1, -4) and (6, 3)

Section 2

a) A line passes through the point (0, 5) and has gradient 2. Write down the equation of this line.
b) Find the gradient and the y intercept of the following lines:
  - i. $y = 4x + 1$
  - ii. $y = 3x - 4$
  - iii. $4x - 3y = 12$
  - iv. $2x + 5y = 20$
  - v. $x - 2y = 0$
c) A line passes through the points with coordinates (1, 3) and (2, 8). Find the equation of the line.
d) The gradient of a line is 3. The point (4, 2) lies on the line. Find the equation of the line.

<u>Section 3</u>

a) Find the equation of a line that is parallel to the line with equation
   $y = 4x – 1$ which passes through the point (0, 3).

b) Find the equation of a line that is parallel to the line with equation
   $2x + y = 4$ which passes through the origin.

c) Find the equation of a line that is perpendicular to the line with equation
   $y = \frac{1}{4}x$ which passes through the point (2, -8).

d) Find the equation of a line that is parallel to the line with equation
   $y + x = 10$ which passes through the point (-2, -5).

<u>Section 4</u>

Work out the length of the line joining each of these pairs of points:

a) (3, 1) and (11, 7)
b) (2, 5) and (12, 29)
c) (-6, 9) and (8, 13)
d) (9, -15) and (-11, 6)
e) A circle has centre point O(4, 2). The point A (9, 14) lies on the circle.

    i) Work out the radius of the circle.

    ii) Which of the following points also lie on the circle?

 (16, 7)   (-1, -10)   (7, 16)   (4, 15)

6. Pythagoras' theorem and trigonometry (videos 257, 259 & 330 – 337)

At GCSE you will have covered Pythagoras in both 2D and 3D and used both right angled trigonometry (SOH CAH TOA) as well as non-right angled trigonometry (sine rule and cosine rules). At A-level this will be built upon mainly by giving the problems a more contextual basis to be solved from.
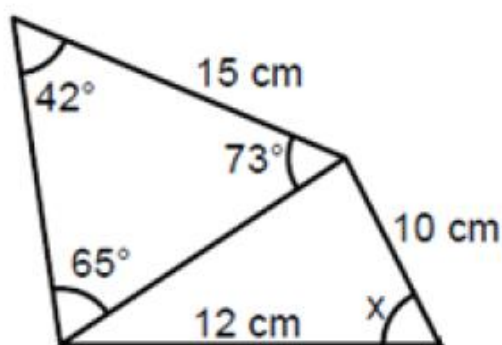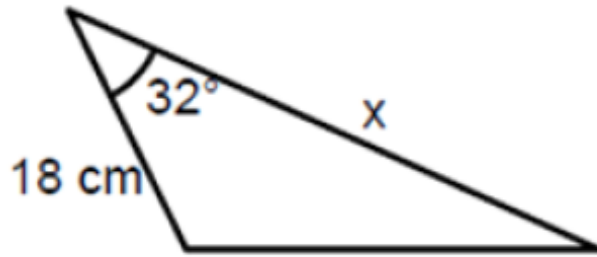
a) Calculate the length of AB.

10.5 cm

15 cm

12 cm

A

B

b) Calculate the length of AB.

B

4 cm
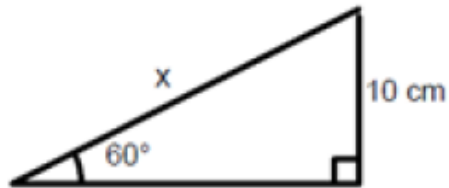
6 cm

A 15 cm
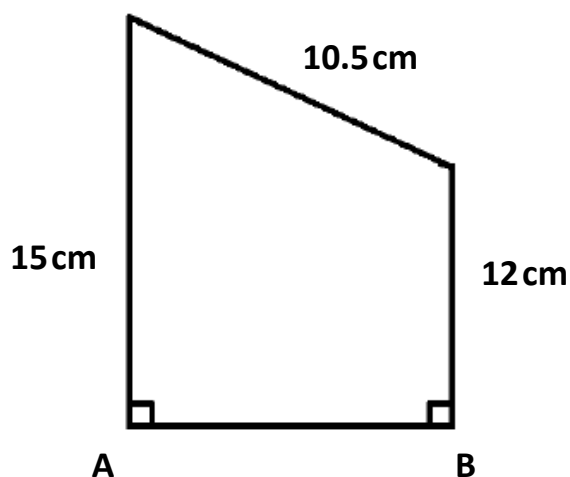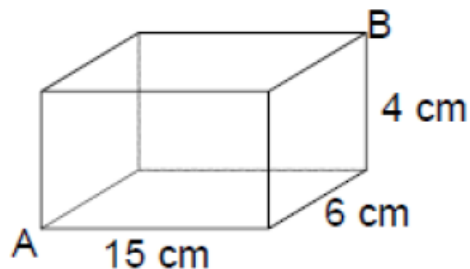
c) Calculate the value of *x*.
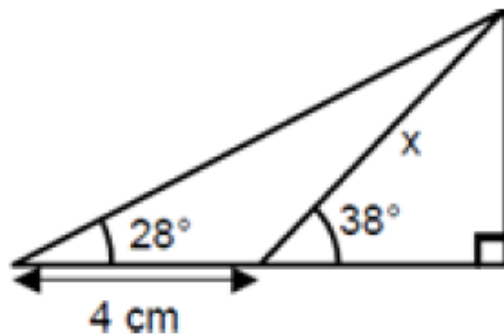
42°

15 cm

73°

10 cm

65°

12 cm

x

d) The area of the triangle is 182.2cm². Find the value of x.

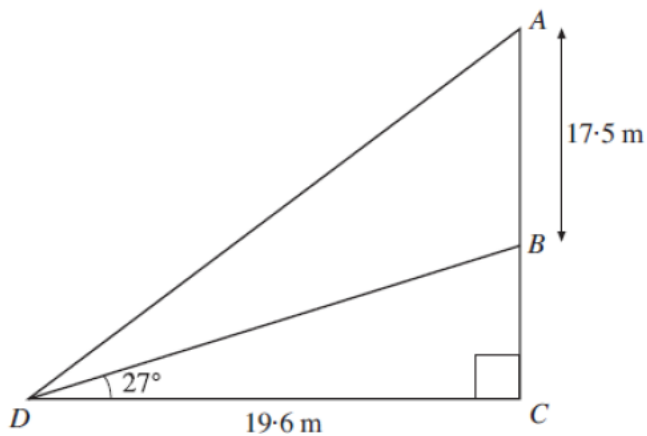e) Calculate the value of x.

f) Find the value of x.

g) In the following diagram, AB = 17.5m, DC = 19.6m, BDC = 27º, DCA = 90º. Calculate the size of angle ADC.

h) Calculate the value of x.